Engineering Notes

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Variable-Displacement Buoys

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THE use of unmanned buoys for acquisition of a variety of remote marine environmental measurements promises to be common in the future.[1] Although buoys offer a cost-effective means of acquiring data, their dynamical motions represent, however, a source of error and noise for some of the desired measurements, e.g., wind vector, gravity, and magnetic field measurements.[2] Consequently, the development of buoys having minimal motion resulting from the wave excitation forces of the sea (highly stable buoys) has become an important engineering problem. This Note discusses the heave motion of a buoy configuration that has a passive variable-displacement capability that results in minimal motion.

The common means of reducing heave motion of floating bodies is to provide a large draft, thereby exploiting the exponential decay of wave excitation forces with water depth. Such a method is the primary cause of stability of the FLIP,[3] SPAR,[4] and POP[4] research platforms. The disadvantage of this method of stabilization is that to obtain greatly reduced heave motion, very large draft is required, resulting in large buoys. The draft of the FLIP, for example, is 300 ft with over 2000 tons displacement.[4] Another method of stabilization is the use of a stepped cross section, which results in an excitation force null upon proper selection of areas and step separation.[4,5] The force null and the consequent heave amplitude null can be achieved, however, only at one wave period, limiting the usefulness of this method. Other stabilization methods such as damping plates and flotation devices are primarily effective only under heave resonance conditions.[5]

It would clearly be desirable to have a buoy configuration that, with limited draft, is highly stable over the complete frequency range of the sea. One approach to the development of such a configuration is the consideration of vibration absorption techniques and the use of two interconnected bodies instead of the common single-body buoy. Such a configuration is shown in Fig. 1. The plate with area A_2 is connected to the main buoy structure by a spring such that the combination resembles a piston-cylinder arrangement. Increases in pressure at the bottom of the plate arising from incoming waves causes compression of the spring and a decrease in the nominal (undisturbed) displacement of the system. Equivalently, the plate absorbs the majority of the wave motion leaving the main body relatively undisturbed. The problem of sealing between the plate and the main body is serious, of course, but for the present we shall disregard this difficulty.

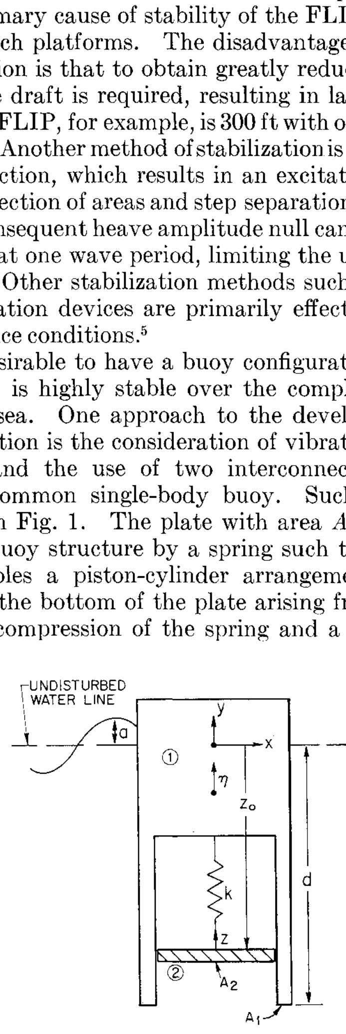

Fig. 1 Variable displacement buoy.

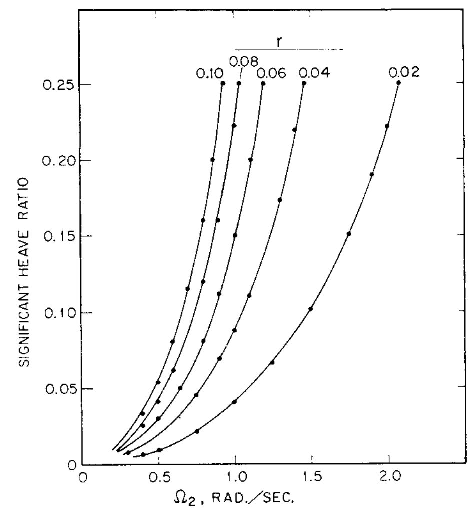

Fig. 2 Ratio of variable-displacement buoy heave to single-body buoy heave.

As shown in Fig. 1, the x,y coordinate system is chosen with origin at the undisturbed waterline, and with the x,y plane containing the center of mass of the complete system. The variables η and z are measures of the vertical displacements of the main body and plate, respectively, relative to the undisturbed waterline. Considering only pure heaving motion in an unmoored condition, the external vertical forces on the bodies are

$$F_1 = -m_1 g - c_1\dot{\eta} + p(-d + \eta)A_1 \quad (1)$$

$$F_2 = -m_2 g - c_2\dot{z} + p(-z_0 + z)A_2 \quad (2)$$

where subscripts 1 and 2 refer to the main body and plate, respectively; m_α is the mass; c_α is the viscous damping coefficient; $p(y)$ is the total pressure at depth y; and A_α is the area. These equations reflect the assumption of viscous damping forces that are independent of the relative positions of the main body and plate and independent of the vertical wave velocity. Such an assumption is only an approximation to the actual situation, but it is hopefully sufficient for investigation of the salient characteristics of the system. The force exerted on the main body from the plate is

$$F_{12} = -F_{21} = c_3(\dot{z} - \dot{\eta}) + k(z_n - z_0 + z - \eta) \quad (3)$$

where c_3 is the viscous friction coefficient for the relative motion between the main body and plate, k is the spring constant, and z_n is the null extension position of the spring. With harmonic deep-water waves, the pressure is given by[6]

$$p(y) = \rho g[-y + ae^{Ky}\cos\omega t] \quad (4)$$

where ρg is the weight density of water, a is the wave amplitude, K is the wave number, ω is the wave frequency; and where the diameter of the system has been assumed small in comparison to the wavelength so that the pressure variation in the x direction can be neglected. Use of Newton's Law provides the equations of motion for the system, which for

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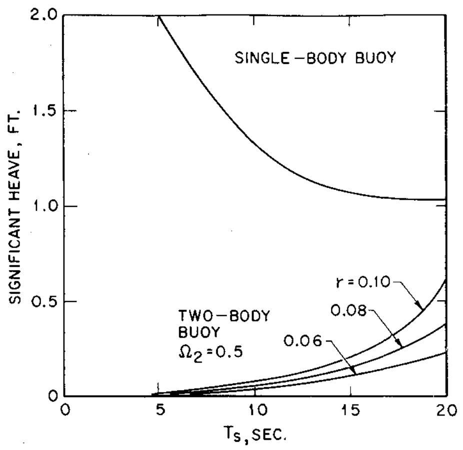

Fig. 3 Significant heave of the two-body and single-body buoy.

quiescent water and equilibrium are

$$m_1 g = \rho g A_1 d + k(z_n - z_0) \tag{5}$$

$$m_2 g = \rho g A_2 z_0 - k(z_n - z_0) \tag{6}$$

The solution of these equations provides the equilibrium draft of the plate,

$$z_0 = (m_1 + m_2)g/\rho g A_2 - (A_1/A_2)d \tag{7}$$

and the nominal compression distance of the spring,

$$(z_n - z_0) = m_1 g/k - (\rho g A_1 d/k) \tag{8}$$

The motion of the system about the equilibrium position in the Laplace transform variable s is described by

$$\begin{bmatrix} B_{11}(s) & B_{12}(s) \\ B_{21}(s) & B_{22}(s) \end{bmatrix} \begin{bmatrix} \eta(s) \\ z(s) \end{bmatrix} = \begin{bmatrix} b\Omega_1{}^2 e^{-Kd} \\ (1/r\Omega_1)^2 e^{-Kz_0} \end{bmatrix} f(s) \tag{9}$$

where

$$B_{11}(s) = s^2 + (2\zeta_1\Omega_1 + r\gamma)s + r\Omega_2{}^2 + b\Omega_1{}^2$$

$$B_{12}(s) = -r(\gamma s + \Omega_2{}^2) \quad B_{21}(s) = -(\gamma s + \Omega_2{}^2)$$

$$B_{22}(s) = s^2 + (2\zeta_2\Omega_2 + \gamma)s + \Omega_2{}^2 + (1/r)\Omega_1{}^2$$

$$r = m_2/m_1 \quad b = A_1/A_2 \quad \Omega_2 = (k/m_2)^{1/2}$$

$$\Omega_1 = (\rho g A_2/m_1)^{1/2} \quad \zeta_1 = c_1/2m_1\Omega_1 \quad \zeta_2 = c_2/2m_2\Omega_2$$

$$\gamma = c_3/m_2 \quad f(t) = a \cos\omega t$$

and where m_1 and m_2 now contain added mass contributions that are assumed constant. An indication of the performance of the system can be obtained by consideration of the undamped natural frequencies. With $\zeta_1 = \zeta_2 = \gamma = 0$ and neglecting the term $b\Omega_1{}^2$ relative to $r\Omega_2{}^2$, the characteristic equation is

$$[s^2 + \Omega_2{}^2 + (1/r)\Omega_1{}^2](s^2 + r\Omega_2{}^2) - r\Omega_2{}^4 = 0 \tag{10}$$

which has the four roots

$$s = \pm j\,\frac{\Omega_2}{[1/r + (1 + r)(\Omega_2/\Omega_1)^2]^{1/2}};$$

$$\pm j[1/r + r(\Omega_2/\Omega_1)^2]^{1/2}\Omega_1 \tag{11}$$

It is seen that small values of the mass ratio r and the spring frequency Ω_2 result in one low system frequency and one high system frequency. By proper selection of these parameters both system frequencies can be placed outside of the bandwidth of the wave spectrum. This also would be expected since a light plate and weak spring enhance the relative absorption capacity of the plate.

The transfer function for the main body heave per foot wave height is

$$\frac{\eta(s)}{f(s)} = G(s) = \frac{B_{22}(s)b\Omega_1{}^2 e^{-Kd} - B_{12}(s)(1/r\Omega_1{}^2)e^{-Kz_0}}{B_{11}(s)B_{22}(s) - B_{12}(s)B_{21}(s)} \tag{12}$$

which is found through elimination of $z(s)$ in Eq. (9). Hence for a wave spectral density $\Phi_a(\omega)$, the main body heave spectrum is $\Phi_\eta(\omega) = |G(j\omega)|^2\Phi_a(\omega)$ and the significant heave S_η is obtained from

$$S_\eta{}^2 = \int_0^\infty |G(j\omega)|^2\Phi_a(\omega)d \tag{13}$$

The ratio of S_η to the significant heave of an identical single-body buoy (i.e., with the plate fixed at $y = z_0$) is shown in Fig. 2 as a function of the spring frequency Ω_2 and the mass ratio r. The significant heave for both buoys is shown in Fig. 3 as a function of significant wave period. These graphs apply to a 63-ton buoy having a draft of 25 ft. The wave spectrum used was the Bretschneider spectrum[7] (at unit significant wave height)

$$\Phi_a(\omega) = 4200/T_s{}^4\omega^5 \quad e^{-1050/T_s{}^4\omega^4} \tag{14}$$

with the significant wave period $T_s = 10$ sec for Fig. 2. The remaining parameter values used for the calculations were $b = 0$, $\Omega_1 = (g/25)^{1/2} = 1.13$ rad/sec, $\zeta_1 = 0$, $\zeta_2 = 0.037$, and $c_3 = 0.5$ lb sec/ft. The damping ratio $\zeta_2 = 0.037$ was also used for calculation of the single-body buoy heave spectrum. This value corresponds to a resonant amplification of five times the wave height for the single-body buoy.

It is seen from these figures that appreciable attenuation of heave motion can be obtained, but the spring frequencies required are very low. For example, with a mass ratio of 0.04 and a heave ratio of 0.1, which requires a spring frequency of 1.05 rad/sec (Fig. 2), the spring stiffness needed is $k = rm_1\Omega_2{}^2 = 173$ lb/ft. Using Eq. (8) (with $d = 0$), the null spring length can be calculated to be an unreasonable 735 ft.

It may be possible, however, to alleviate the engineering problems associated with the low spring stiffness requirement through the use of air springs, which are particularly well-suited for low frequency systems. If the cavity between the plate and the main body contained air (entrapped by a ring-stiffened bellows, possibly), the spring constant would be given by[8]

$$k = A_2(m_1 + m_2)\gamma g/V_0 \tag{15}$$

where $\gamma = 1.4$ and V_0 is the equilibrium volume of entrapped air. The volume required to obtain a small k could be provided in a hollow main body structure serving as a reservoir for the air supply. For the example considered above, the reservoir volume would have to be 40 times the volume of the plate cavity A_2z_0 to give a spring constant of 173 lb/ft. The equilibrium air pressure under these conditions would be 11 psi (gage). Such a mechanism may be possible for floating platform legs, and, although there are many engineering problems associated with the configuration that have not been enumerated herein, may be a realizable method of acquiring highly stable motion over the complete sea spectrum without the need of large draft.

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